MHD instabilities in stellar radiative regions - a linear study -

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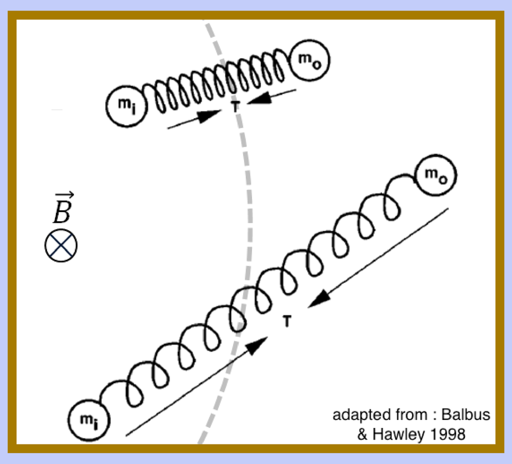
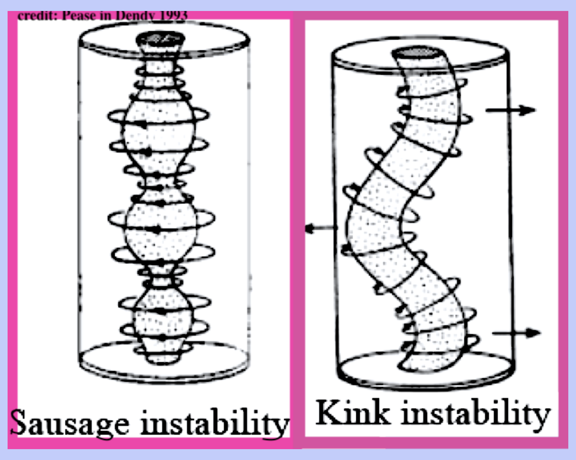
Introduction

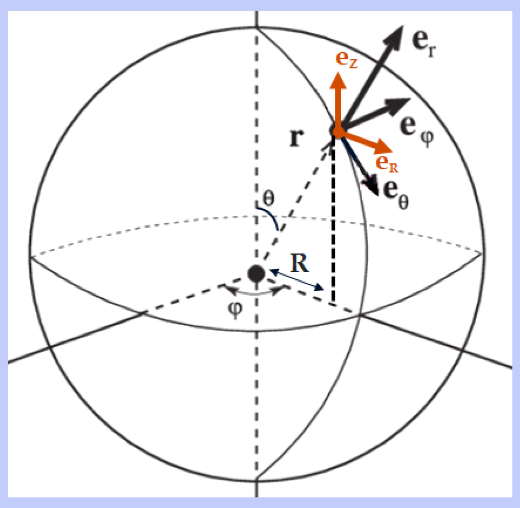
Magnetohydrodynamic instabilities are potentially able to play a key role in the angular momentum transport inside radiative regions of stars. We focus our work on Magneto-Rotational Instability (MRI) and Pinch or Tayler Instability (TI) for which a comprehensive view of these instabilities in stellar interiors is needed. **We aim to understand the role of MHD instabilities in stellar evolution and dynamo**, thus we need to first find out the condition for them to arise.

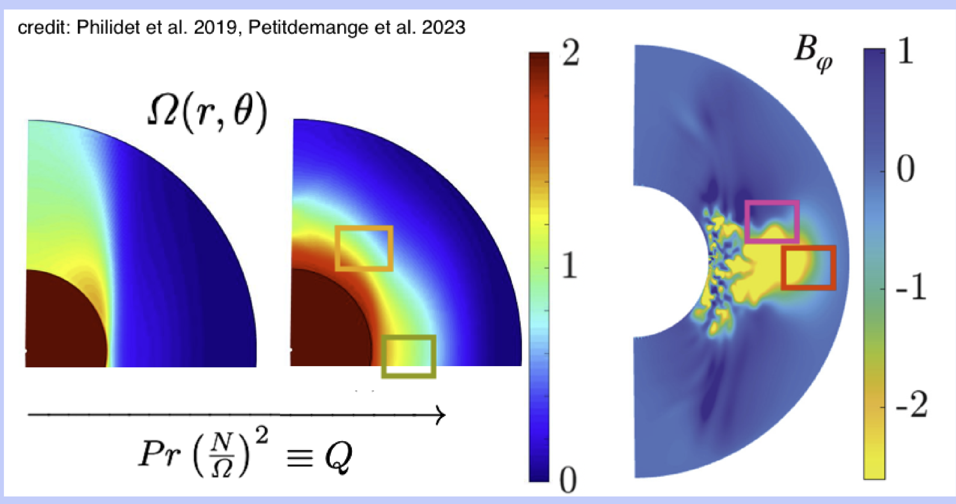




While the MRI corresponds to the destabilization of the rotation profile through magnetic tension, the TI is the wrapping of the magnetic field due to its topology.

We consider a plasma with a background velocity $U_0 = R\Omega(R,Z)$, a magnetic field $B_0 = B_\phi(R,Z) + B_Z$, and a temperature background $T_0(r)$, in a cylindrical frame.





MHD & Thermal equations

$$\partial_t U + U.\nabla U = -\frac{1}{\rho}\nabla P + \frac{1}{\rho\mu}(\nabla\wedge B)\wedge B - \rho g + \nu\Delta U$$

$$\partial_t B + U.\nabla B = B.\nabla U + \eta\Delta B$$

$$\partial_t T + u.\nabla T = \kappa\Delta T \quad \text{with } \rho(T) = \rho(1-\alpha T)$$

Considering perturbations as :

$$A = A_0 + a' \text{ with } a' \propto a e^{i(k_r R + m\phi + k_z Z) + \sigma t}$$

We set our eigen-problem as $\mathcal{H}v = \sigma v$ [1], with $v = (u'_r, u'_\phi, b'_r, b'_\phi, \theta')$

Notations

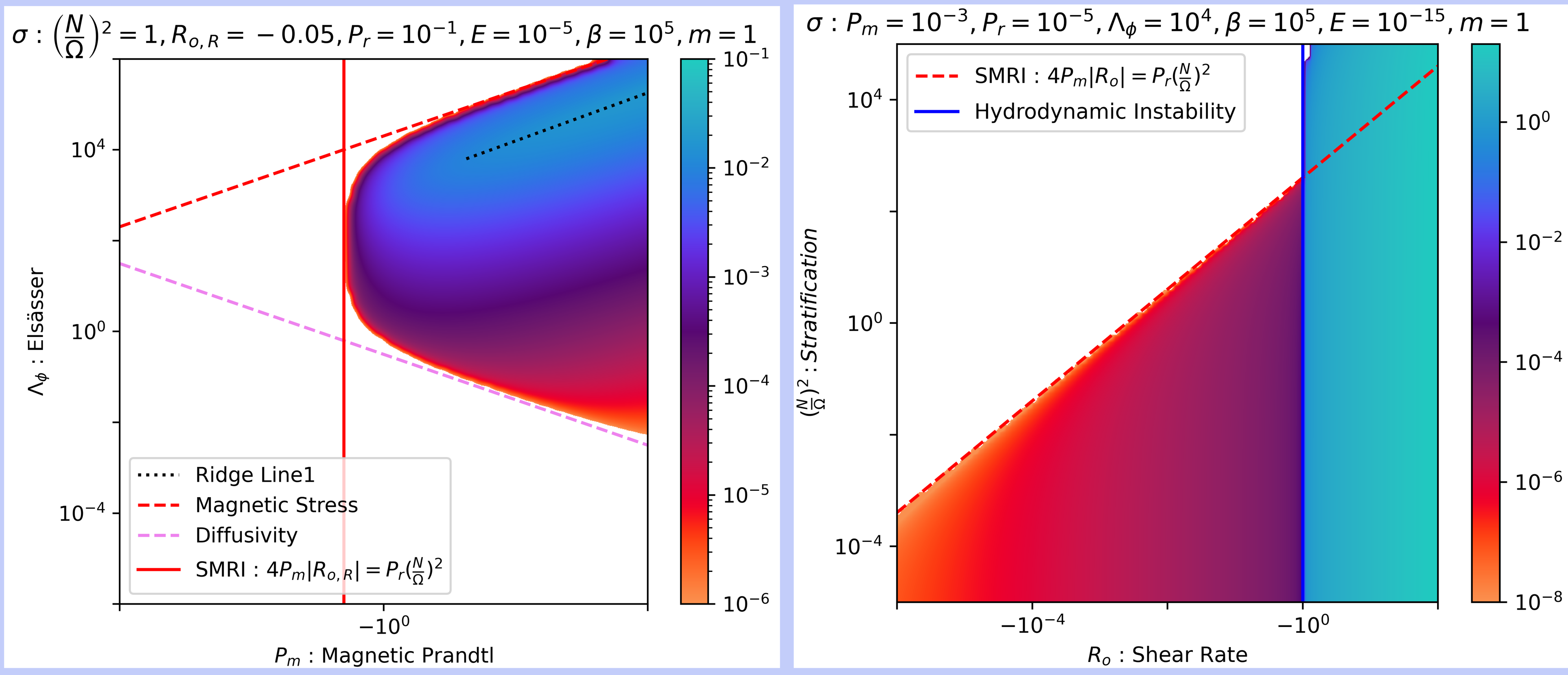
Scaling parameters and their values estimated in radiative regions and taken for simulations :

Parameter	Ratio	Rad.	Simu.
Prandtl	$P_r = \frac{\nu}{\kappa}$	10^{-5}	10^{-1}
Magnetic Prandtl	$P_m = \frac{\nu}{\eta}$	10^{-3}	1
Ekman	$E = \frac{\nu}{\Omega L^2}$	10^{-15}	10^{-5}
Elsässer	$\Lambda = \frac{B^2}{\mu\rho\Omega\eta}$	1	1
Stratification	$\frac{N^2}{\Omega^2} = \frac{\alpha g \partial T}{\Omega^2 \partial r}$	10^3	10

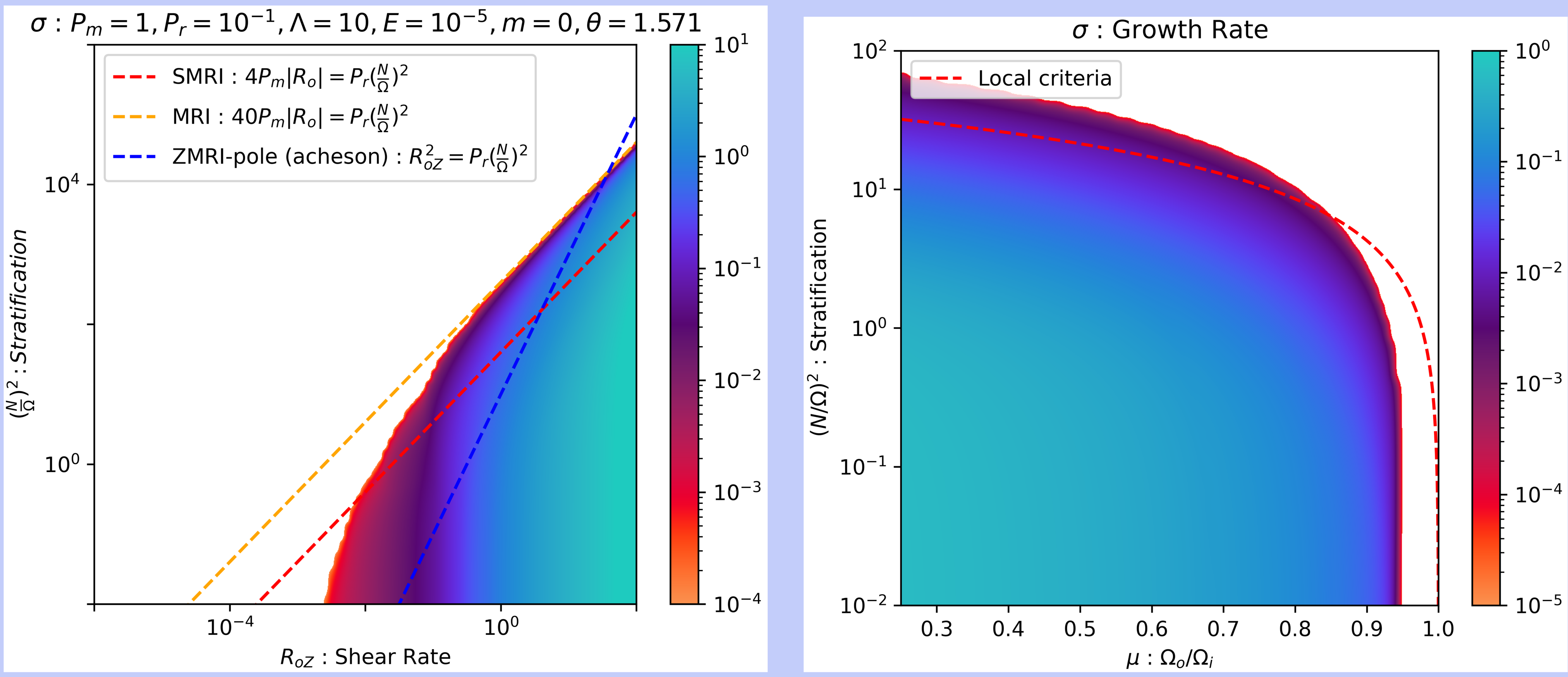
We note : $\alpha = \frac{k_Z}{k}$, $R_{o,R} = \frac{R}{2\Omega}\frac{\partial\Omega}{\partial R}$, $\beta = \frac{B_\phi}{B_Z}$
$R_{b,R} = \frac{R^2}{2B_\phi}\frac{\partial B_\phi/R}{\partial R}$ and similarly for $R_{o,Z}, R_{b,Z}$

Azimuthal-MRI and Axial Shear

We first recover the **Standard MRI criteria** [2]: $P_r\left(\frac{N}{\Omega}\right)^2_c = 4P_m|R_{o,R}|$, and we exhibit the limit due to the magnetic stress $\frac{\Lambda E m^2}{P_m}$ and diffusion $\frac{\Lambda P_m m^2}{E}$ more noticeable for the simulation parameters.



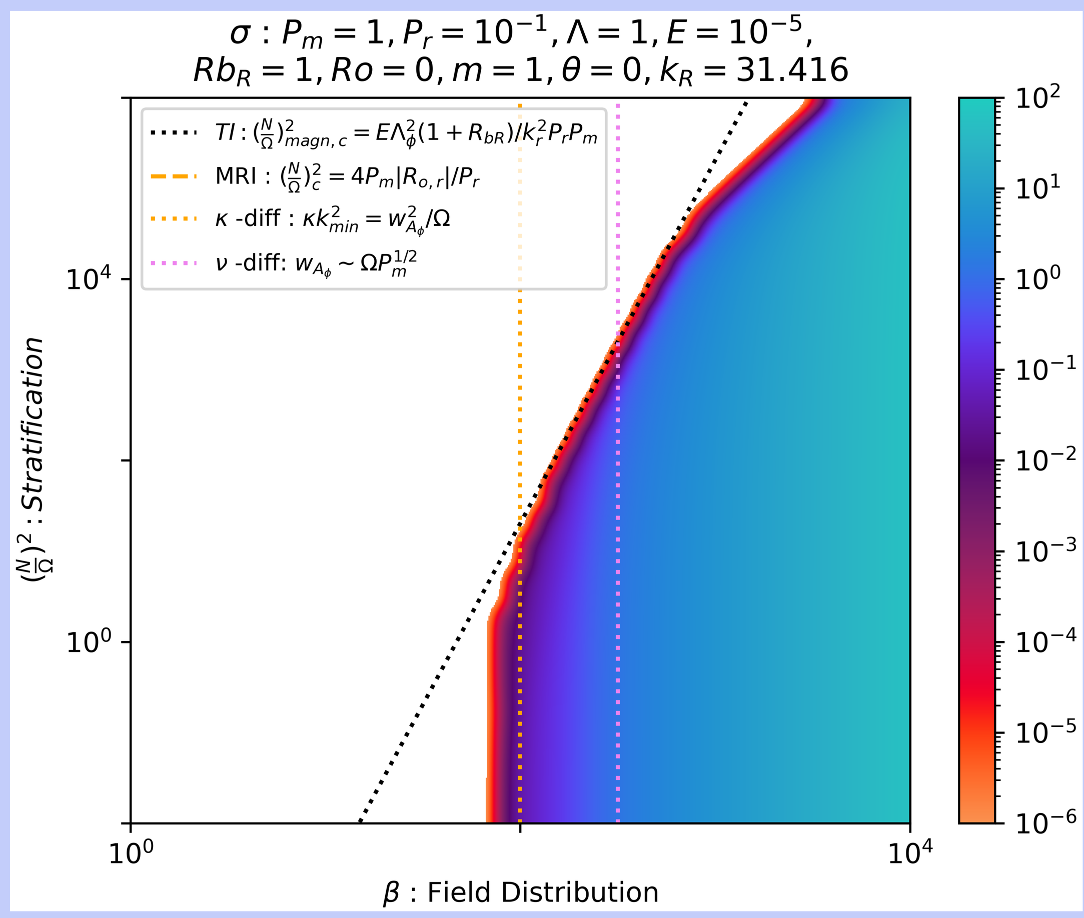
For $|R_{o,R}| > 1$ the flow is hydrodynamically unstable, this corresponds to the Rayleigh limit. The growth rate $\sigma \sim w_{A\phi}\sqrt{|R_{o,R}|}$ while $w_{A\phi} < \sqrt{|R_{o,R}|}\Omega$. Making the AMRI more sensitive to the magnetic field than the SMRI. When an **Axial Shear** $\Omega(Z)$ is considered, the mode $\alpha = k_Z/k$ has the best compromise bewteen the magnetic tension $\sim\alpha$, and the shear destabilization $\sim 1/\alpha^2 - 1$.



A global study on a Taylor-Couette MHD stratified flow is in progress to understand how the instability criteria evolve from a local to a global perpective. This project is being conducted in collaboration with Prof. R. Hollerbach.

Tayler instability

At the pole, considering a toroidal field $B_\phi(R)$ can lead to the **Tayler Instability** if its gradient is steep enough. The stability criterion is:
$P_r\left(\frac{N}{\Omega}\right)^2 > \frac{E\Lambda_\phi^2}{2P_m k_R^2}$ which is reachable in DNS and radiative stellar regions
At a latitude Θ, the modes $\alpha = cos\Theta$ are unaffected by the stratification, making an even wider unstable domain. **At the equator**, only the Magnetic Buoyancy Instability (MBI) remains with the criteron: $P_r\left(\frac{N}{\Omega}\right)^2 > 4E\Lambda_\phi(1+R_{b,r})$ In both cases, the growth rate $\sigma \sim w_{A\phi}^2/\Omega$ for weakly magnetized flow and $\sigma \sim w_{A\phi}$ for the strongly ones.



Conclusion & Outlook

This preliminar work starts to enhance a better understanding of MHD instabilities and highlighted the importance of the toroidal field. A more complete study needs to be carried out to compare the different instability criteria. The global approach will shed light on the importance of curvature terms and boundary conditions. At the end, DNS will be confronted to this linear approach.

References

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