Hidden magnetic fields in stellar interiors probed by asteroseismology

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Introduction

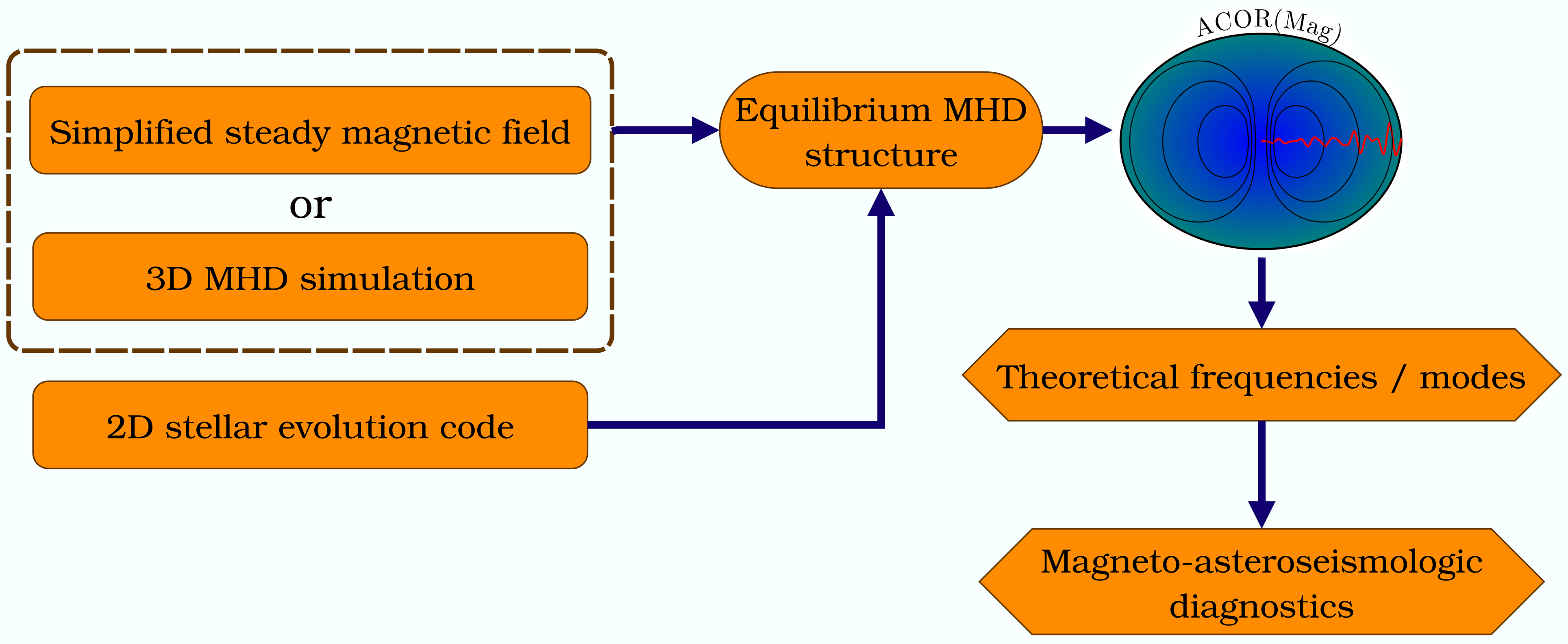
Understanding the distribution and evolution of angular momentum is essential for understanding **stellar formation and evolution.** Recent missions like CoRoT and *Kepler* have highlighted **flaws in current angular momentum transport models.**[1,2] Magnetic field is a promising candidate to explain the missing transport, and hopefully solve the issue. Asteroseismology via perturbative methods has recently enabled the **first detection and measurements of magnetic fields in stellar interiors.**[3] Here, we present an **upgrade** of the **non-perturbative oscillation code ACOR**[4,5] that will **fully incorporate stellar rotation and magnetic fields in a two-dimensional framework**. This code will be able to test the existing perturbative methods and tackle the area of strong and complex internal magnetic fields, thereby allowing realistic seismic diagnostics of magnetic fields.

ACOR(Mag)

ACOR(Mag) is a code based on **ACOR** (Adiabatic Code of Oscillation including Rotation) that computes **2D adiabatic non-radial pulsations of a rotating star in a non-perturbative way**, taking into account the **centrifugal deformation** and the **full Coriolis acceleration**. The goal is to add the **effects on the oscillations induced by the magnetic field**.

Assumptions :

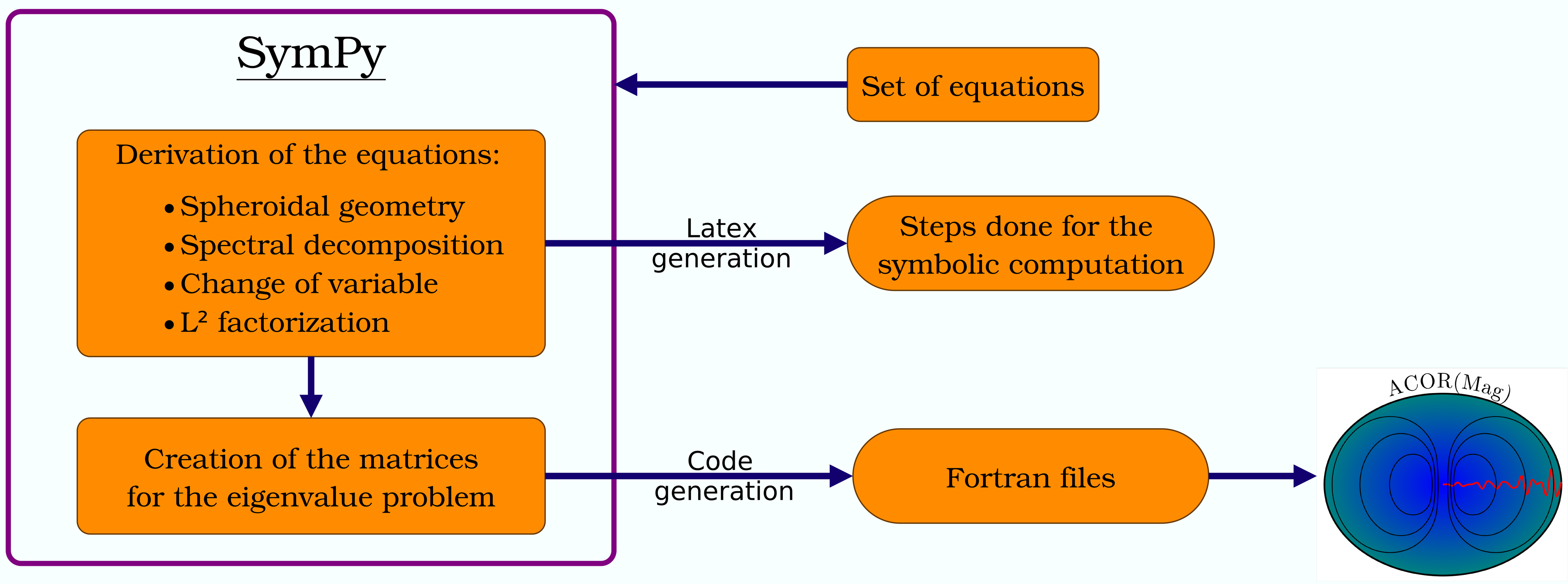
- The star's hydrostatic structure is axissymetric
- The magnetic field is aligned with the rotation axis
- The deformation of the structure due the to magnetic field is neglected



Code	ACOR	$\Longrightarrow$	ACOR(Mag)
Radially differentiated variables over the total number	4/7		8/10
Separation of the Eigenvalue problem for each parity	separation		no separation

Automations with SymPy

Due to the **complexity of the problem** involving **MHD equations**, **spectral decomposition** using spherical harmonics, and **spheroidal coordinates**, formulating the equations proved to be more challenging than anticipated. To overcome these difficulties, I decided to use **SymPy**, a Python library for symbolic computation, and successfully **automated** the various processes required for **deriving the set of equations**.



Sanitary check ($\vec{B_0} = \vec{0}$) $\Longrightarrow$ **ACOR** vs **ACOR(Mag)** same frequencies up to $10^{-14}\mu$Hz

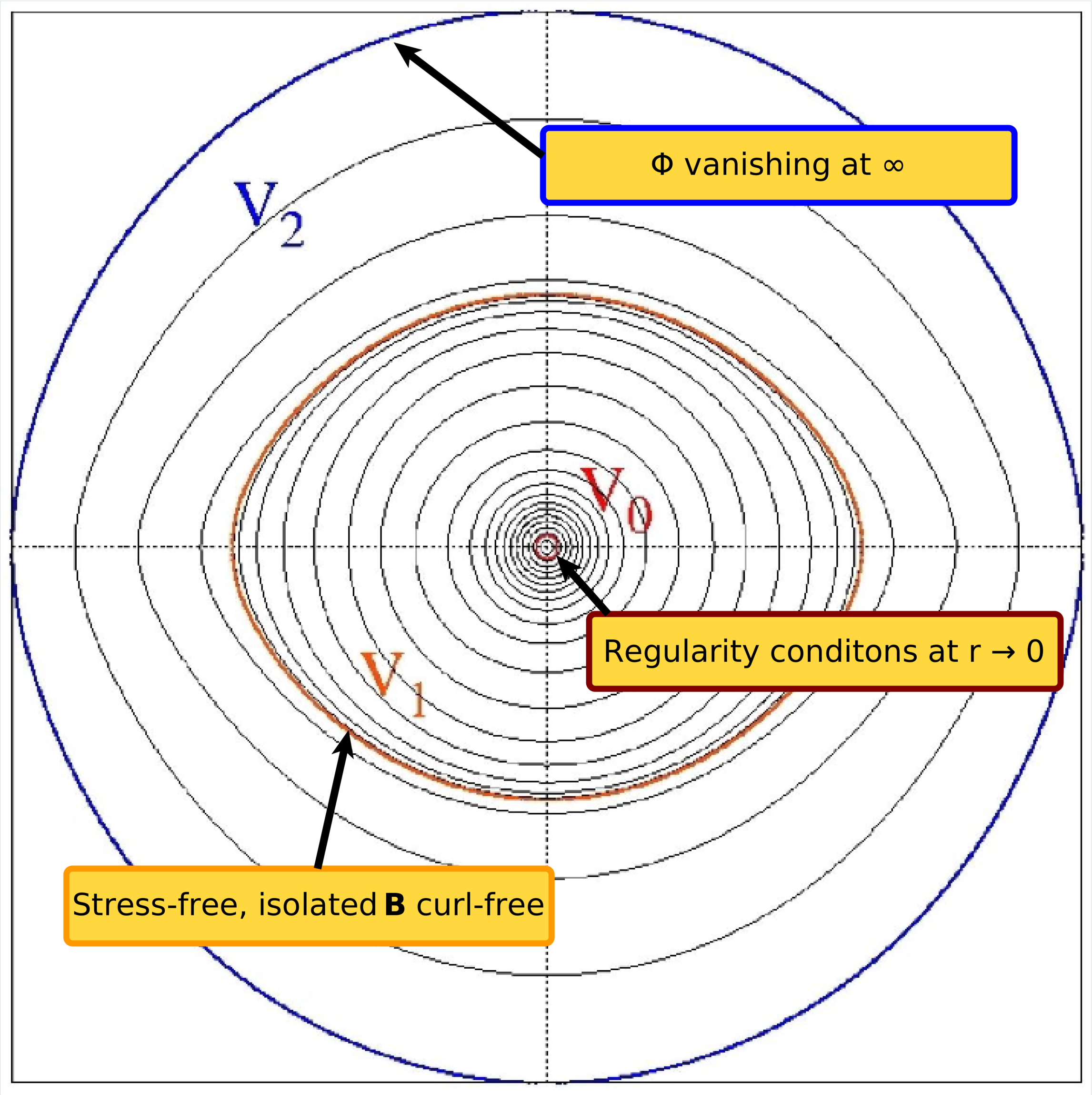
Advantages :

- **Greater flexibility** and **adaptability** (set of equations taken as an input)
- **Faster analytical developments** and **code implementation** in the long run
- **Improved error control**

Conclusion and outlook

We described the initial steps of a 2D oscillation code that includes full effects induced by the rotation and magnetic field. The routines developed using SymPy have enabled **faster analytical developments** and **code implementation**, while also **improving error control**. This will make it easier to **test the assumptions** made by **current methods**[7,8] for **different topologies**. Then, it is planned to **tackle the regime of strong fields** and to **compare the theoretical frequencies** with the **observed** ones from the Kepler mission to develop **magneto-asteroseismologic diagnostics**.

Geometry



We use **spheroidal coordinates**[5] to **avoid discontinuities** and **facilitate the implementation of the boundary conditions**. The system of coordinates varies from spherical at the center (V_0) to spheroidal following the star's deformation (V_1) until it reaches the surface, returning to spherical (V_3) and matching an outer sphere far from the star.

MHD oscillation equations

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}\cdot(\rho \boldsymbol{v}) = 0$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla}\right)\boldsymbol{v} + \frac{\boldsymbol{\nabla} p}{\rho} + \boldsymbol{\nabla}\Phi - \frac{1}{4\pi\rho}(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B} = 0$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla}\right)\left(\frac{\delta\rho}{\rho} - \frac{1}{\Gamma_1}\frac{\delta p}{p}\right) = 0$$

$$\Delta\Phi - 4\pi G\rho = 0$$

$$\frac{\partial \boldsymbol{B}}{\partial t} - \boldsymbol{\nabla}\times(\boldsymbol{v}\times\boldsymbol{B}) = 0$$

For : $X = X_0(r,\theta) + X'(r,\theta,\phi,t)$
X' and δX, Eulerian and Lagrangian perturbation, and $\boldsymbol{v_0} = \boldsymbol{\Omega}\times\boldsymbol{r}$

Numerical method

- **Spectral decomposition** on M spherical harmonics (8M differentiated over 10M variables)
- **Radial discretization** : 4th-order finite differences scheme[6]
- **Solving eigenvalue problem : Newton-like method**, solving the system for the Eigenfunction with an initial guess σ_0, producing a deviation $\delta\sigma$ and iterating until convergence

$$\frac{dy_1}{d\zeta} = (A_{11} + \delta\sigma A_{12})y_1 + (A_{21} + \delta\sigma A_{22})y_2$$

$$0 = (B_{11} + \delta\sigma B_{12})y_1 + (B_{21} + \delta\sigma B_{22})y_2$$

References

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[6] Scuflaire et al. 2008, Ap&SS, 316:83-91
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[8] Dhouib et al. 2022, A&A, 661:A133