

The influence on the internal transport processes when an adjacent zone is added in stellar physics

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Aims

1. Long term project: What are the interactions occurring at the interface between the Convective Zone (CZ) and the Radiative Zone (RZ)?
2. Short-term project: Studying the effect of the (RZ) stratification on the heat flux distribution at the surface of a convective shell for a low-mass star.
3. Mid-term project: Studying the energy transferred from the (CZ) to the (RZ).

Introduction

The internal processes which occur between the (RZ) and the (CZ) in the stars are still unclear and need to be unveiled. 3D numerical simulations allow to investigate the roles played by the different physical mechanisms such as the internal waves, the turbulence, the heat. We will present some (preliminary) results about the effect of the addition of a (RZ) below a (CZ) on the heat flux distribution at the low mass stars surface.

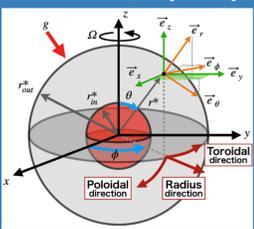
3D spherical model of a rotating star:

- ▶ The hydrodynamical equations:
 - ▶ Navier-Stokes, mass conservation & energy
- ▶ Assumptions:
 - ▶ Decompose the thermodynamical variables $X = (S, T, P)$ into a reference state \tilde{X} and a convective fluctuation X' ; *i.e.* $X = \tilde{X} + X'$; such that $\{P', T'\} \ll \{P, T\}$ and $\tilde{S} \sim S'$
 - ▶ The \tilde{X} is a polytropic solution; *i.e.* hydrostatic equation in (close to) an adiabatic stratified atmosphere.
 - ▶ Anelastic approximation (LBR form): \Leftrightarrow (a) density variations are allowed for the reference state; (b) the sound waves are filtered out.
 - ▶ Adiabatic \Leftrightarrow (a) Isentropic reference state ($dU = dQ + dW = TdS + dW \Rightarrow dS = 0$) (b) no heat transfer/heat's source (frictions); *i.e.* all the internal energy is transformed into a mechanical one
- ▶ Mean: The code MagIC [4]
- ▶ To model the zones: (CZ) $\Leftrightarrow \nabla_r \tilde{S} \leq 0$ and (RZ) $\Leftrightarrow \nabla_r \tilde{S} \geq 0$

I. Equations [1] [2] [6] [7] [8] and stars considered [5] [8]

Low mass stars considered:

- ▶ Young red giant stars with a thick convective shell ($r \in [0.35, 1]r_o$)
- ▶ Solar-like stars with a thin convective shell ($r \in [0.8, 1]r_o$)



Extracted from [9]

Navier-Stokes, mass, energy & reference states equations under their dimensionless form equations in the anelastic approximation:

- ▶ Navier-Stokes:

$$\frac{Dv}{Dt} = -\frac{1}{E_k} \nabla \left(\frac{P'}{\rho} \right) - 2\frac{1}{E_k} \Omega \wedge v - \frac{Ra}{Pr} S'g + \frac{1}{\rho} \nabla \cdot S$$
- ▶ Where S is the Strain tensor
- ▶ Mass conservation:

$$\nabla \cdot (\tilde{\rho}v) = 0$$
- ▶ Energy (Entropy form):

$$\frac{DS'}{Dt} + v_r \nabla S' = \frac{1}{Pr \tilde{\rho} T} \nabla \cdot (\tilde{\rho} \tilde{T} \nabla S') + \frac{1}{\tilde{\rho} T} \frac{Pr}{Ra Di} Q_v$$

- ▶ Where Q_v is the viscous heating

Non adiabatic reference states :

- 2/ Solving these equations numerically

1/ Imposed the entropy gradient:

- ▶ Solar-like star [8] :

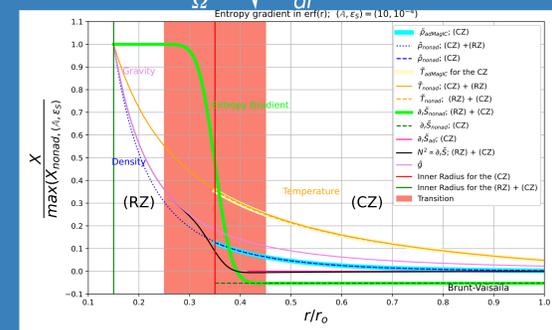
$$\frac{\partial \tilde{S}}{\partial r} = A \left\{ 1 + \tanh(\zeta(r_B - r)) \right\}$$
- ▶ Young red giant star:

$$\frac{\partial \tilde{S}}{\partial r} = A \left\{ 1 + \operatorname{erf}(\zeta(r_B - r)) \right\}$$
- ▶ With the Dissipation number Di (compares the thermal length and the shell length, a high Di means a high density contrast) and the **adiabatic deviation**

$$\epsilon_S = \left. \frac{d}{dr} \left(\frac{d\tilde{S}}{dr} \right) \right|_{r_o}$$

Characteristic values in the (CZ):

| Parameter | Ratio | IRL. | Simu. |
|------------------|--|------------|--------------------------------|
| Prandtl | $Pr = \frac{\nu}{\kappa}$ | 10^{-5} | {0.1, 0.5} |
| Rayleigh | $Ra = \frac{G \Delta S d^3}{c_P \nu \kappa}$ | 10^{30} | $10^6 - 10^9$ |
| Ekman | $E_k = \frac{L_{conv}}{\Omega d^2}$ | 10^{-15} | $\{10^{-5}, 3 \cdot 10^{-4}\}$ |
| Density contrast | $N_\rho = \log(\tilde{\rho}_i / \tilde{\rho}_o)$ | / | {1.1, 4, 5} |
| Polytrop. index | n | ? | 2 |
| Nusselt | $Nu = 1 + \frac{L_{conv}}{L_{rad}}$ | / | / |
| Brunt-Väisälä | $N^2 = \frac{1}{c_P} \frac{\partial \tilde{S}}{\partial r} \tilde{g}$ | / | / |
| Conv. Rossby | $Ro_c = \sqrt{\frac{Ra E^2}{Pr}}$ | / | / |
| (RZ) strength | $\frac{N}{\Omega} \propto \sqrt{\left(\tilde{g} \frac{d\tilde{S}}{dr} \right) _{r_i}}$ | / | [2.55-14.2] |



Backgrounds for a thick (CZ) and a thin (RZ)

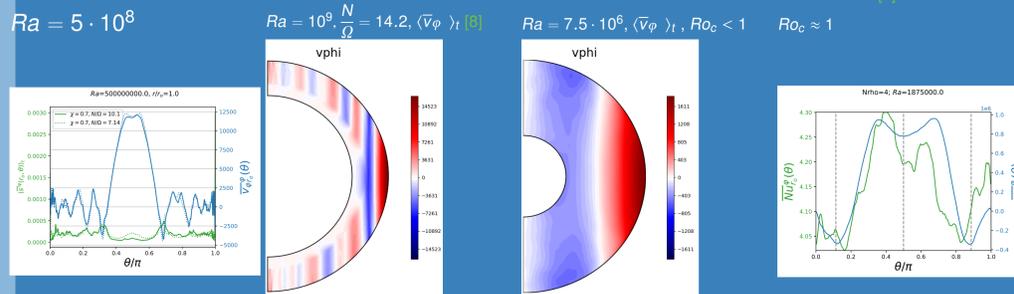
II. Numerical Methods [1][3][4]

- ▶ Spherical geometry \Rightarrow Variables are decomposed by using the:
 - ▶ Chebyshev polynomials or Finite differences in the radial direction r .
 - ▶ Spherical harmonic functions for the angle components θ & ϕ .
- ▶ v is a solenoidal field ($\nabla \cdot (\tilde{\rho}v) = 0 \Rightarrow v$) can be decomposed into:
 - ▶ a poloidal component v_P .
 - ▶ a toroidal component v_T .
 - $\Rightarrow v = \nabla \wedge \nabla \wedge (v_P r) + \nabla \wedge (v_T r) \Rightarrow v = \left(\frac{1}{\tilde{\rho}} \frac{\partial}{\partial r} \left(\tilde{\rho} \frac{\partial v_P}{\partial r} \right) + \frac{1}{\tilde{\rho}} \frac{\partial}{\partial r} \left(\tilde{\rho} v_T \right) \right) / (r, \theta, \phi)$ With L_2 the Beltrami Laplacian.
 - ▶ v has 3 unknowns and depends on only 2 scalar fields v_P & v_T . The radial component is purely radial.
- ▶ Linear terms solved in the spectral space *a contrario* of the non linear ones and the Coriolis force.
- ▶ Mixed explicit/implicit scheme for the time integration (Adam Bashforth scheme).

III. Influence of the (RZ) on the internal transport processes in the (CZ)

Solar-like stars, (RZ) and (CZ), $Ro_c \approx 0.45$

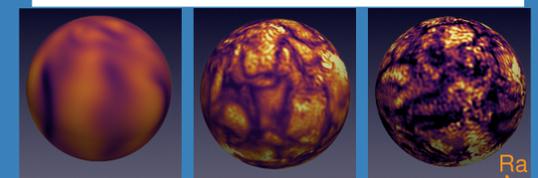
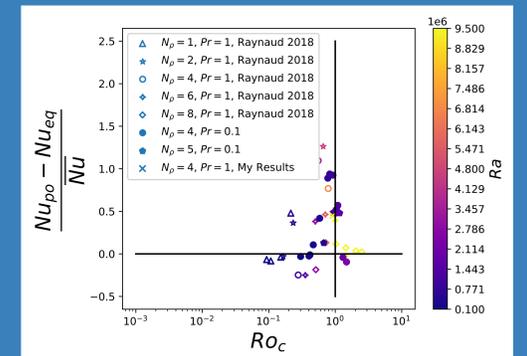
Young red giant star, (CZ) only. $Pr = 0.1$. From a solar to an anti-solar model as in [5]



Summary

- Right column: As in [5] but for a higher value of the thermal diffusivity κ , therefore a more realistic Prandtl number Pr ; when the convective Rossby number is greater than 1, then the prograde jet at the equator becomes a retrograde one which is decoupled from the heat flux distribution at the surface.
- ▶ The heat distribution is homogenized because the angular momentum is well-mixed due to the turbulent convective motions.
- Left column :
 - ▶ Reproduction of one of the models from in [5] (right figure).
 - ▶ The heat flux distribution and the zonal flow increases at the surface but only at the equator (left figure) when the stratification strength of the (RZ) decreases N/Ω . A small value of N/Ω means that the (CZ) penetrates deeper the (RZ).
 - ▶ It proves that some exchanges occur between the two zones. In a close future, we will quantify these exchanges.

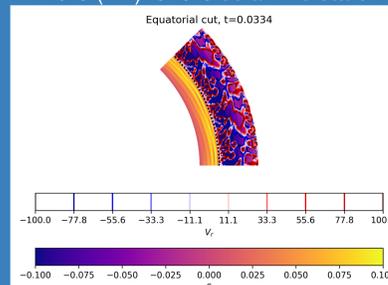
The heat flux distribution as a function of the convective Rossby number



Entropy fluctuations at $r = 0.99 r_o$

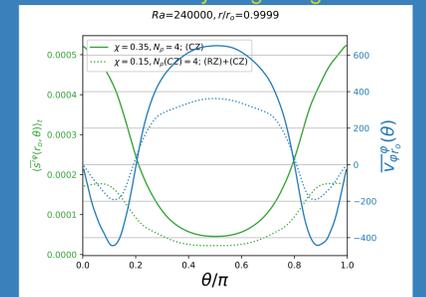
IV. Perspectives: Energy transferred from the (CZ) to the (RZ)

Influence of the (CZ) on the waves propagation in the (RZ) for the solar-like stars :



- ▶ The convective plumes pummel the (RZ) and excited the internal waves
- ▶ Short project: Finding the (gravito)-inertial waves by using the methods in [12] [13]
- ▶ Long-term project: Changing the dynamic of the overshooting [11] by changing the thermal diffusivity κ and see how the energy is transferred from the (CZ) to the (RZ).

Influence of the (RZ) on the heat distribution at the surface for the young red giant stars:



- ▶ For a moderate turbulent flow, the anti-correlation is maintained ($Ro_c \approx 0.45$), but the amplitude of the entropy fluctuations and the zonal velocity decrease when the (RZ) is added.
- ▶ Aim: Is the homogenization of the heat flow accelerated by the presence of the (ZR)? (*i.e.* at a given N/Ω , we need to have $Ro_c \geq 1$?).

Conclusions

1. In a single (CZ), the heat flux distribution at the surface is controlled by the convective Rossby number. The heat is homogenized at the surface when the buoyancy is stronger than the Coriolis force.
2. Preliminary results: it seems that the adding of a (RZ) allows to accelerate the heat flux distribution at the surface.
3. At a given level of turbulence, by decreasing the degree of stratification in the (RZ), the anti-correlation is maintained for the zonal velocity and the heat flux distribution at the surface but their both amplitudes are increasing at the equator (but it is not the case anymore at the other latitudes).

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