Heat flux and zonal flow distributions at the surface of a solar-like star
Modeling simultaneously the Radiative and the Convective zones

Nairabeze Alexandre[†] , Belkacem Kévin[†] and Petitdemange Ludovic[†]

[†]LIRA, Observatoire de Paris, PSL University

Aims

1. Short term: See the evolution of the heat flux distribution and the zonal flow in a Convective Zone (CZ) at the surface of a rotating star with a more realistic parameter.
2. Mid term: Repeat the short term project but with a Radiative Zone (RZ) below the (CZ). To do that, I have to implement it in the code MagIC.
3. Long term : See more carefully the interactions at the interface

Introduction

A star is ball of gas whose the heat produced by the nuclear reactions has to be evacuated from the core to the surface. Two main means of heat transport co-exist: the convection and the radiation which give the name of these regions: the Convective Zone (CZ) and the Radiative Zone (RZ). Both regions are more or less understood, but there is still some work to describe properly the interface.

3D spherical model of a rotating star:

- The equations:
  - Navier-Stokes, mass conservation & energy
- Assumptions:
  - Decompose the thermodynamical variables $X = (S, T, P)$ into a reference state $\tilde{X}$ and a convective fluctuation X' ;*i.e.* $X = \tilde{X} + X'$; such that $\{P', T'\} << \{\tilde{P}, \tilde{T}\}$ and $\tilde{S} \sim S'$
  - The $\tilde{X}$ is a polytropic solution; *i.e.* hydrostatic equation in (close to) an adiabatic stratified atmosphere.
  - Anelastic approximation: $\iff$ (a) density variations are allowed for the reference state; (b) the sound waves are filtered out.
  - Adiabatic $\iff$ (a) Isentropic reference state ($dU = dQ + dW = TdS + dW = dW \Rightarrow dS = 0$) (b) no heat transfer/heat's source (frictions); *i.e.* all the internal energy is transformed into a mechanical one
- Mean: The code MagIC [4]
- To model the zones: (CZ) $\iff \nabla_r\tilde{S} \leq 0$ and (RZ) $\iff \nabla_r\tilde{S} \geq 0$

I. Navier-Stokes, mass, energy & reference states equations [1] [2] [6] [7] [8]

Dimensionless equations (obtained with the shell radius and the viscous time as the characteristic values):

- Navier-Stokes & mass conservation:

$$\frac{D\mathbf{v}}{Dt} = -\frac{1}{E_k}\nabla\left(\frac{P'}{\tilde{\rho}}\right) - 2\frac{1}{E_k}\Omega \wedge \mathbf{v} - \frac{\mathrm{Ra}}{\mathrm{Pr}}S'\mathbf{g} + \frac{1}{\tilde{\rho}}\nabla \cdot \mathrm{S}$$

$$\nabla \cdot (\tilde{\rho}\mathbf{v}) = 0$$

- Where S is the Strain tensor :

$$S_{ij} = 2\nu\tilde{\rho}\left[e_{ij} - \frac{1}{3}\delta_{ij}\nabla \cdot \mathbf{v}\right],$$

$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right),$$

- Energy:

$$\frac{DS'}{Dt} + v_r\nabla\tilde{S} = \frac{1}{Pr\tilde{\rho}\tilde{T}}\nabla \cdot \left(\tilde{\kappa}\tilde{\rho}\tilde{T}\nabla S'\right) + \frac{1}{\tilde{\rho}\tilde{T}}\frac{\mathrm{Pr}}{\mathrm{RaDi}}Q_\nu$$

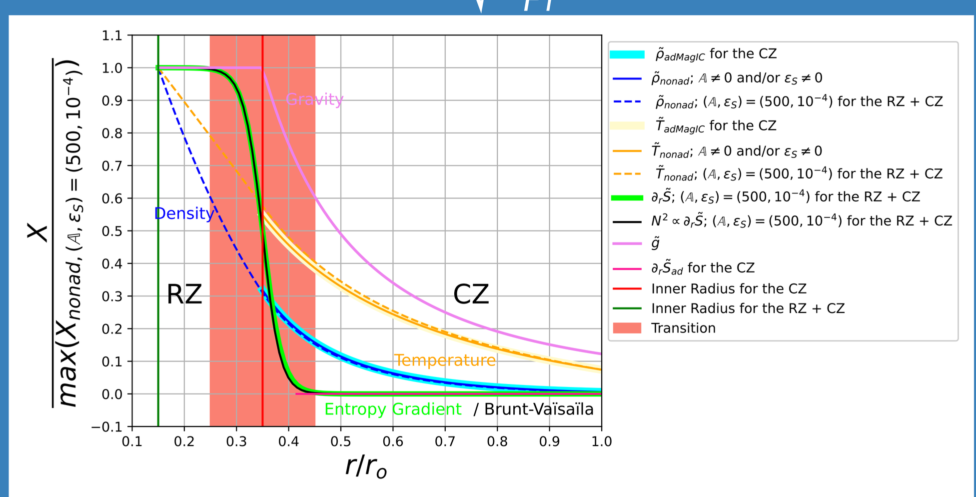
- Where Q_ν is the viscous heating :

$$Q_\nu = \sigma_{ij}\frac{\partial v_i}{\partial x_j},$$

$$\sigma_{ij} = \tilde{\rho}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\nabla \cdot \mathbf{v}\right),$$

- And σ_{ij} the stress tensor.

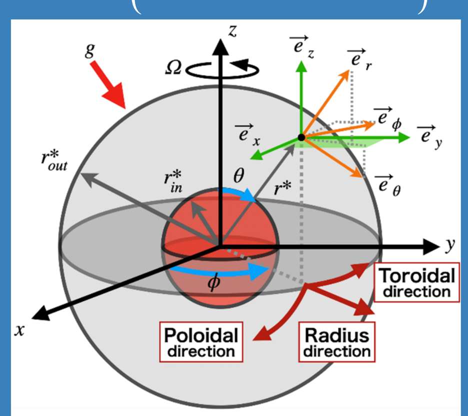
Parameter	Ratio	IRL.	Simu.
Prandtl	$Pr = \frac{\nu}{\kappa}$	10^{-5}	10^{-1}
Rayleigh	$Ra = \frac{G\Delta S d^3}{c_P\nu\kappa}$	10^{30}	10^{3}
Ekman	$E_k = \frac{\nu}{\Omega d^2}$	10^{-15}	$3 \cdot 10^{-4}$
Stratification	$N_\rho = \log(\tilde{\rho}_i/\tilde{\rho}_o)$	/	$\{4, 5\}$
Polytrop. index	n	?	2
Nusselt	$Nu = \frac{L_{rad} + L_{conv}}{L_{rad}}$	/	/
Brunt-Vaïsaïla	$N^2 = \frac{1}{c_P}\frac{\partial\tilde{S}}{\partial r}\tilde{g}$	/	10^4
Local Rossby	$Ro_l(r_o) = v_{RMS}(r_o)/(\Omega l(r_o))$	/	1
Conv. Rossby	$Ro_c = \sqrt{\frac{RaE^2}{Pr}}$	/	/



Non adiabatic reference states :

- Imposed the entropy gradient:

$$\frac{\partial\tilde{S}}{\partial r} = \mathbb{A}\left\{1 + \tanh\Big(\zeta(r_B - r)\Big)\right\}$$



Extracted from [9]

- With the Maxwell relations, $\tilde{P} = \tilde{\rho}\Re\tilde{T}$and $\nabla\tilde{P} = \tilde{\rho}\tilde{\mathbf{g}}$, I have partially solved analytically these dimensionless equations:
-

$$\frac{1}{\tilde{T}}\frac{d\tilde{T}}{dr} = \epsilon_S\frac{d\tilde{S}}{dr} - \frac{\mathrm{Di}}{\tilde{T}}\tilde{g}(r) \Rightarrow \tilde{T}(r) = e^{\epsilon_S\tilde{S}(r)}\left(C_1 - \mathrm{Di}\int_{r_i}^{r} e^{-\epsilon_S\tilde{S}(r)}\tilde{g}(r)dr\right)$$

$$\frac{1}{\tilde{\rho}}\frac{d\tilde{\rho}}{dr} = -\epsilon_S\frac{d\tilde{S}}{dr} - \frac{n}{\tilde{T}}\mathrm{Di}\tilde{g}(r) \Rightarrow \tilde{\rho}(r) = \tilde{T}^n\exp\left\{\epsilon_S\tilde{S}\right\}^{-(n+1)}$$

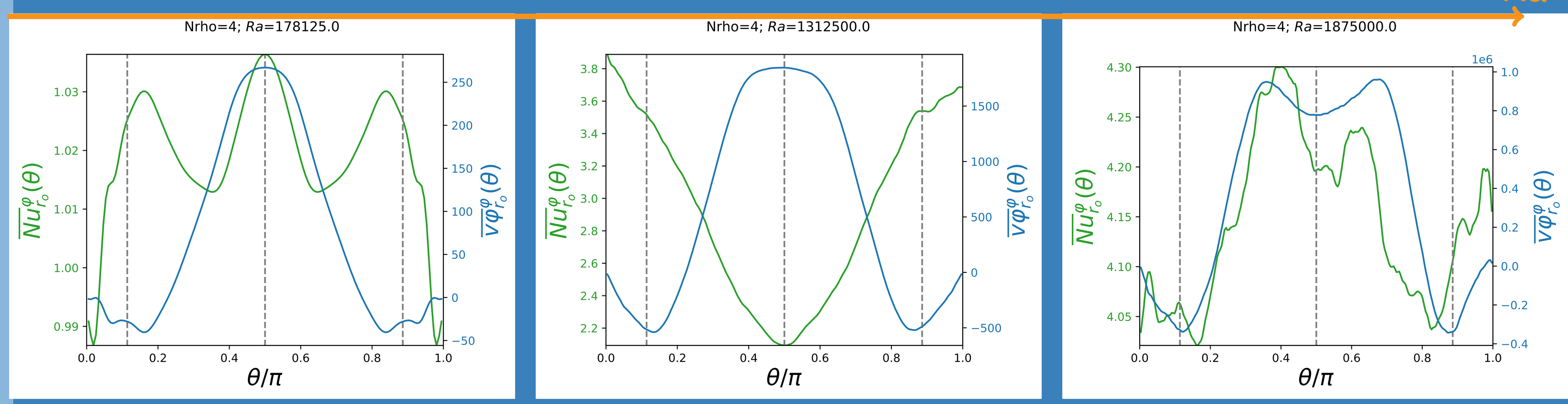
- With the Dissipation number $\mathrm{Di} = \frac{1 - e^{\left\{\frac{N_\rho}{n}\right\}}e^{\left\{\frac{\epsilon_S(\tilde{S}_i - \tilde{S}_o)}{n}\right\}}}{-\int_{r_i}^{r_o} e^{-\epsilon_S\tilde{S}(r)}\tilde{g}(r)dr}e^{-\epsilon_S\tilde{S}_o}$; $C_1 = \frac{1}{e^{\epsilon_S\tilde{S}(r_o)}} + \mathrm{Di}\int_{r_i}^{r_o} e^{-\epsilon_S\tilde{S}(r)}\tilde{g}(r)dr$ and the adiabacity deviation $\epsilon_S = \frac{d}{c_P}|\frac{dS}{dr}|_{r_i}$

II. Numerical Methods[1][3][4]

- Spherical geometry $\Rightarrow$ The variables are decomposed by using the:
  - Chebyshev polynomials or Finite differences in the radial direction r.
  - Spherical harmonic functions for the angle components θ & φ .
- $\mathbf{v}$ is a solenoidal field($\nabla \cdot (\tilde{\rho}\mathbf{v}) = 0 \iff \mathbf{v}$) can be decomposed into:
  - a poloidal component v_P.
  - a toroidal component v_T.
  - $\Rightarrow \mathbf{v} = \nabla \wedge \nabla \wedge (v_P\mathbf{r}) + \nabla \wedge (v_T\mathbf{r}) \Rightarrow \mathbf{v} = \begin{pmatrix} \frac{1}{r}L_2(v_P) \\ \partial_\theta(\frac{1}{r}\partial_r(rv_P)) + \frac{1}{\sin\theta}\partial_\varphi v_T \\ \frac{1}{\sin\theta}\partial_\varphi(\frac{1}{r}\partial_r(rv_P)) - \partial_\theta v_T, \end{pmatrix}_{(r,\theta,\varphi)}$ With L_2 the Beltrami Laplacian.
  - $\mathbf{v}$ has 3 unknowns and depends on only 2 scalar fields v_P & v_T. The radial component is purely radial.
- Linear terms solved in the spectral space *a contrario* of the non linear ones and the Coriolis force.
- Mixed explicit/implicit scheme for the time integration (Adam Bashforth scheme).

III.(a) Heat flux distribution Nu, zonal flow v_φ at the star's surface in a (CZ)

- Simulations made at E_k, n, N_ρ fixed; increasing the turbulence through the Ra and at a more realistic parameter $Pr = 0.1$. Below, $N_\rho = 4$.



Heat flux distribution Nu

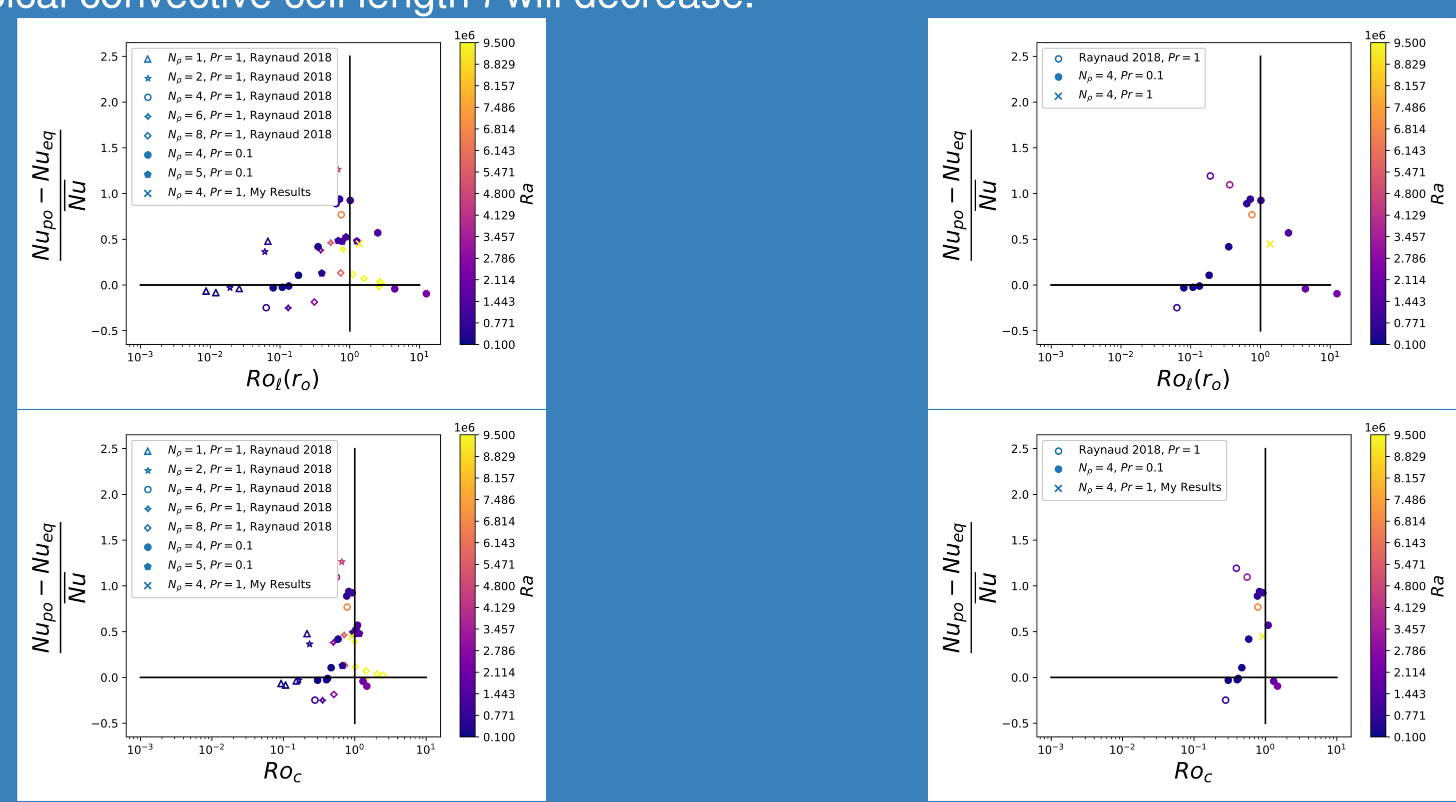
- Close to the convection's onset, the heat flux distribution is constant
- If the Ra is increasing, there is an anti-correlation between Nu and v_φ . At the equator the heat flux is very well mixed (convection & prograde jet).
- If the turbulences are too strong, the Nu becomes uniform.

Zonal flow v_φ [10]

- Low $Ra \Rightarrow$, Prograde jet at the equator due to a high rotation rate $\Omega \iff$ weak E_k. Convective cells have a "banana cylinders shape". aligned with the rotation axis
- Because, the Coriolis force dominates the buoyancy (if $Ro_c > 1$)
- If Ra is too high, at the equator a retrograde jet is formed (the buoyancy dominates) because the angular momentum is homogenized. It explains the decoupling.
- The bands of retro/prograde jets are due to $Ro_c(r)$.

III.(b) Influence of the Prandtl number on the Nu in a (CZ) [5]

- Raynaud [5] at $Pr = 1$; a particular value (the viscous and the thermal time have the same weight).
- According to him, the heat flux contrast ΔNu is higher when the $Ro_l(r_o) \in [0.1, 1]$ (when the Coriolis forces dominates the inertia) and collapse sharply when the inertia dominates.
- I have verified it with a lower Prandtl ($Pr = 0.1$) which means that κ increases and the typical convective cell length l will decrease.



- At $Pr = 0.1$, $N_\rho = 4$; and even at $Pr = 1$, $N_\rho = 4$, there is not a sharp collapse at $Ro_l(r_o) > 1$ *a contrario* of the $Ro_c(r_o)$.
- By decreasing the Pr, we can see the amplitude of the heat flux distribution is weaker. It is due to the decreasing of l.
- It explains also the horizontal shift observed for the $Ro_l(r_o)$ at $N_\rho = 4$.

Conclusions

1. It seems that by decreasing the Prandtl number, the physical parameter which could explain the collapse of the heat flux distribution at the surface could be the convective Rossby number Ro_c and not the local Rossby one $Ro_l(r_o)$ as suggested by Raynaud.

Perspectives

1. Adding a (RZ) below the (CZ) and study its influence on the Nu and the v_φ .
2. See also the influence of the radiative zone's size (through r_b) , the slope of the transition (through ζ) in the $\nabla\tilde{S}$, ...
3. See the spherical modes which are excited at the surface with one or 2 zones.
4. Long term: Instead to use this approach to model the interface (through $\nabla\tilde{S}$), I would like to take in account carefully the interactions which occur.

References

[1] Jones et *al.*, *Anelastic Convection-Driven Dynamo Benchmarks.* Icarus, 10.1016/j.icarus.2011.08.014, 2011
[2] Mizerski, *Rigorous Entropy Formulation of the Anelastic Liquid Equations in an Ideal Gas.* JFM, 10.1017/jfm.2017.723, 2017
[3] Glatzmaier, *Numerical simulations of stellar convective dynamos. I. the model and method.* Journal of Computational Physics, 10.1016/0021-9991(84)90033-0, 1984
[4] MagIC, https://magic-sph.github.io/
[5] Raynaud et *al.*, *Gravity Darkening in Late-Type Stars: I. The Coriolis Effect.* A&A, 10.1051/0004-6361/201731729, 2018
[6] Gastine et *al.*, *Stable Stratification Promotes Multiple Zonal Jets in a Turbulent Jovian Dynamo Model.* Icarus, 10.1016/j.icarus.2021.114514, 2021
[7] Dietrich et *al.*, *Penetrative Convection in Partly Stratified Rapidly Rotating Spherical Shells.* Frontiers in Earth Science, 10.3389/feart.2018.00189, 2018 Frontiers in Earth Science, 10.3389/feart.2018.00189, 2018
[8] Wulff et *al.*, *Zonal Winds in the Gas Planets Driven by Convection above a Stably Stratified Layer.* Monthly Notices of the Royal Astronomical Society, 10.1093/mnras/stac3045, 2022
[9] Satake et *al.*, *Influence of Centrifugal Buoyancy in Thermal Convection within a Rotating Spherical Shell.* Symmetry, 10.3390/sym14102021, 2022
[10] Gastine et *al.*, *Zonal Flow Regimes in Rotating Anelastic Spherical Shells: An Application to Giant Planets.* Icarus, 10.1016/j.icarus.2013.02.031, 2013

Contact Information

- E-mail: alexandre[dot]nairabeze[at]obspm.fr
- Website of the team: https://dynamos.obspm.fr/index.html